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MATHEMATICAL MODELLING AND SIMULATION FRAMEWORK OF A DQ-FRAME CONTROLLED SHUNT ACTIVE POWER FILTER WITH NONLINEAR HYSTERESIS REGULATION

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ABSTRACT

This paper presents a rewritten research-paper version of the mathematical modelling chapter for a shunt active power filter (SAPF) controlled through a synchronous dq-frame instantaneous power strategy and nonlinear hysteresis current regulation. The proposed framework connects a voltage source inverter in shunt with a three-phase nonlinear load at the point of common coupling. Harmonic and reactive current components are extracted through Clarke and Park transformations, while the DC-link voltage loop supports the active current component required for inverter losses and voltage stabilization. A nonlinear hysteresis regulator then forces the filter current to track the generated reference current with adaptive switching boundaries. The model includes the three-phase source, diode-rectifier nonlinear load, coupling inductor, DC-link capacitor, voltage source inverter, dq-frame controller, reference-current generation, switching logic, and performance indices such as current THD, voltage THD, power factor, tracking error, DC-link voltage deviation, settling time, and filter efficiency. The resulting formulation provides a compact simulation-ready basis for evaluating harmonic mitigation and power-quality enhancement under nonlinear loading conditions.

Keywords: Shunt active power filter[1], harmonic compensation[2], dq-frame control[3], nonlinear hysteresis current regulation[4], power quality[5], MATLAB/Simulink[6].

1. INTRODUCTION

Converter-fed loads, controlled rectifiers, adjustable-speed drives, battery chargers, and switched-mode power supplies have made modern distribution systems more controllable, but they also draw non-sinusoidal current from the grid. The resulting harmonic currents increase feeder losses, heat transformers and machines, reduce power factors, disturb protection and measurement devices, and may push the point of common coupling beyond accepted harmonic limits. For this reason, harmonic compensation is now a core requirement in power-quality-oriented distribution networks. Passive tuned filters can attenuate selected harmonic orders, yet their behaviour is tied to fixed component values and network impedance. They may also introduce resonance when the system condition changes. Active power filters overcome these restrictions by measuring the distorted load current and injecting a controlled compensating current that cancels the unwanted components in real time. The shunt active power filter is particularly suitable for current harmonic mitigation because it is connected in parallel with the load and directly controls the source current profile. The modelling framework developed in this paper combines dq-frame harmonic extraction with nonlinear hysteresis current control. In the synchronous reference frame, fundamental components appear as nearly constant quantities, whereas harmonic components appear as oscillatory terms. This property simplifies filtering and reference generation. The nonlinear hysteresis regulator improves the final current-tracking stage by adapting the hysteresis band according to operating conditions, which helps limit switching-frequency variation while retaining fast transient response.

2. PROPOSED SAPF CONFIGURATION

The proposed system consists of a balanced three-phase utility source, a nonlinear diode-rectifier load, a voltage source inverter, a DC-link capacitor, interface inductors, a dq-frame instantaneous power controller, and a nonlinear hysteresis current regulator. The SAPF is placed at the point of common coupling so that the nonlinear load and the compensator share the same grid connection node. The load current is separated conceptually into a fundamental active component and unwanted harmonic and reactive components. The inverter is commanded to produce the latter components locally. When the injected current matches the reference, the grid no longer supplies the harmonic or reactive demand; it supplies mainly the fundamental active current required by the load.

$$i_s(t) = i_L(t) - i_f(t)$$
$$i_L(t) = i_1(t) + i_h(t)$$
$$i_f^*(t) = i_h(t) + i_q(t)$$

Here, i_s is the source current, i_L is the nonlinear load current, i_f is the actual filter current, i_f^* is the reference filter current, i_1 is the fundamental active current component, i_h is the harmonic current component, and i_q is the reactive current component. Under ideal compensation, $i_f(t)$ approaches $i_f^*(t)$, and the source current reduces to the fundamental active component.

$$i_s(t) \approx i_1(t)$$

3. MATHEMATICAL MODEL

3.1 THREE-PHASE SOURCE MODEL

The grid is represented as a balanced three-phase sinusoidal supply. For a 415 V, 50 Hz system, the phase voltages are expressed as follows:

$$v_a(t) = V_m \sin(\omega t), \quad v_b(t) = V_m \sin\left(\omega t - \frac{2\pi}{3}\right), \quad v_c(t) = V_m \sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$\omega = 2\pi f, \quad V_{ph} = \frac{V_{LL}}{\sqrt{3}}, \quad V_m = \sqrt{2}V_{ph}$$

With $V_{LL} = 415$ V and $f = 50$ Hz, the RMS phase voltage is approximately 239.6 V, the peak phase voltage is approximately 338.85 V, and the angular frequency is 314.16 rad/s. Harmonic distortion is therefore not imposed at the source-voltage level in the base model; it is produced by the nonlinear load current and then mitigated by the SAPF.

3.2 NONLINEAR LOAD MODEL

The nonlinear load is modelled as a three-phase diode bridge rectifier feeding a resistive load. This configuration is widely used for active-filter studies because it produces discontinuous current pulses and dominant low-order harmonics. The current drawn by the rectifier can be represented by a Fourier expansion:

$$i_L(t) = I_1 \sin(\omega t + \phi_1) + \sum_{n=5}^{\infty} I_n \sin(n\omega t + \phi_n), \quad n = 6k \pm 1$$

For a six-pulse rectifier, the dominant current harmonics occur at the 5th, 7th, 11th, 13th, 17th, and 19th orders. These components raise the RMS current without contributing useful active power, which lowers the operating power factor and increases distribution losses. The SAPF control objective is therefore to identify these harmonic terms and synthesize an opposing current through the inverter.

3.3 SAPF POWER CIRCUIT MODEL

The SAPF power circuit comprises a three-phase voltage source inverter connected to the PCC through filter inductors. The inductor limits the current ripple caused by inverter switching and shapes the injected current. For each phase, the compensating current dynamics can be written in compact form as:

$$L_f \frac{di_f}{dt} = V_f - V_s$$

where L_f is the coupling inductance, V_f is the inverter output voltage, and V_s is the corresponding phase source voltage. The DC side of the inverter is supported by a capacitor whose stored energy is

$$E_{dc} = \frac{1}{2} C_{dc} V_{dc}^2$$

The DC-link voltage must remain sufficiently high and stable so the inverter can generate the commanded AC compensation voltage. A voltage-control loop is therefore included to regulate V_{dc} around its reference value and compensate converter losses.

4. DQ-FRAME INSTANTANEOUS POWER CONTROL

The control algorithm begins by transforming measured three-phase load currents from the abc frame into the stationary $\alpha\beta$ frame using the Clarke transformation. The stationary variables are then converted into synchronously rotating dq variables using the Park transformation.

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}$$

The angle θ is obtained from a phase-locked loop synchronized with the utility voltage. In the dq frame, the average components correspond to the fundamental active and reactive quantities, while harmonic components appear as ripple. Low-pass filters are used to estimate the average values, and the harmonic components are recovered by subtraction:

$$i_{dh} = i_d - \bar{i}_d, \quad i_{qh} = i_q - \bar{i}_q$$

The DC-link voltage controller adds the active current required to hold the capacitor voltage at its reference. If $e_{dc} = V_{dc}^* - V_{dc}$, the controller contribution may be represented as

$$i_{loss} = K_p e_{dc} + K_i \int e_{dc}\, dt$$

The final reference currents are generated in dq form and transformed back to abc coordinates. These references command the SAPF to inject the harmonic current, reactive current, and small active component needed for DC-link voltage regulation.

5. NONLINEAR HYSTERESIS CURRENT REGULATION

The nonlinear hysteresis current regulator is the inner current-control layer. It compares the measured filter current with the reference current produced by the dq-frame controller. The instantaneous tracking error is

$$e(t) = i_f^*(t) - i_f(t)$$

In a fixed-band hysteresis controller, switching occurs when $e(t)$ crosses constant upper or lower limits. The nonlinear regulator replaces this constant band with an adaptive band that depends on the DC-link voltage, coupling inductance, desired switching frequency, and reference-current slope:

$$h_n = \frac{V_{dc}}{8 f_{sw} L_f}\left[1 - \left(\frac{2L_f}{V_{dc}}\frac{di_f^*}{dt}\right)^2\right]$$

The switching rule for each inverter leg is then expressed as

$$S = 1 \quad \text{when} \quad e(t) < -h_n, \qquad S = 0 \quad \text{when} \quad e(t) > h_n$$

This adaptive boundary reduces unnecessary high-frequency switching when the current slope changes and improves current tracking during dynamic conditions. As $e(t)$ approaches zero, the actual filter current approaches its reference value and the source current approaches the fundamental active component.

6. MATLAB/SIMULINK IMPLEMENTATION

The simulation model is organized into six interacting subsystems: source, nonlinear load, voltage source inverter, DC-link capacitor, dq-frame controller, and nonlinear hysteresis regulator. Source voltage, load current, filter current, and DC-link voltage are measured continuously. The controller processes these signals to generate reference compensation currents, and the hysteresis block converts the current error into inverter switching commands.

Table I. Simulation parameters used for the dq-frame controlled SAPF

S. No.	Parameter	Symbol	Value	Unit
1	Source line voltage	V_s	415	V
2	System frequency	f_s	50	Hz
3	Nonlinear load resistance	R_L	10	Ω
4	Filter inductance	L_f	5	mH
5	DC-link capacitance	C_{DC}	2200	μF
6	DC-link reference voltage	V_{DC}	700	V

The 415 V, 50 Hz source represents a standard low-voltage three-phase distribution system. The 10 Ω rectifier load produces a harmonic-rich current profile. A 5 mH coupling inductor limits switching ripple and shapes the compensation current. The 2200 μF DC-link capacitor provides energy buffering, while the 700 V DC-link reference gives the inverter sufficient voltage margin to synthesize the required compensating current.

7. Performance Evaluation Indices

The proposed model is intended for quantitative assessment using harmonic, dynamic, and control-quality indicators. Current THD is the primary index for SAPF performance because the shunt filter is designed to prevent nonlinear-load harmonic current from entering the source.

$$THD_I = \frac{\sqrt{I_2^2 + I_3^2 + \cdots + I_n^2}}{I_1} \times 100\%$$

$$THD_V = \frac{\sqrt{V_2^2 + V_3^2 + \cdots + V_n^2}}{V_1} \times 100\%$$

$$HCR = \frac{THD_{before} - THD_{after}}{THD_{before}} \times 100\%$$

Power-factor improvement is evaluated from the relationship between active power and apparent power. Current-tracking quality is measured through instantaneous error, mean-square error, and root-mean-square error. DC-link regulation is assessed from the voltage deviation relative to the reference value. Settling time after load disturbance and filter efficiency are also useful for judging dynamic response and converter utilization.

$$PF = \frac{P}{S} = \cos\phi$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[i_f^*(k) - i_f(k)\right]^2}$$

$$\Delta V_{dc}(\%) = \frac{|V_{dc}^* - V_{dc}|}{V_{dc}^*} \times 100\%$$

$$\eta = \frac{P_{comp,useful}}{P_{filter,total}} \times 100\%$$

Together, these indices allow the SAPF to be judged not only by harmonic reduction but also by current-tracking accuracy, DC-link stability, transient performance, and operating efficiency. Compliance-oriented evaluation should compare the compensated current spectrum with the applicable limits of IEEE Std. 519.

8. CONCLUSION

A research-paper-format modelling framework has been developed for a dq-frame controlled shunt active power filter using nonlinear hysteresis current regulation. The SAPF is connected in parallel with a diode-rectifier nonlinear load and commanded to inject the harmonic and reactive current components that would otherwise be drawn from the grid. The mathematical model covers the source, nonlinear load, inverter interface, DC-link capacitor, dq-frame transformations, reference-current generation, nonlinear hysteresis switching logic, and evaluation indices. The dq-frame controller provides a convenient mechanism for separating average fundamental components from oscillatory harmonic components, while the DC-link voltage loop sustains the inverter energy balance. The nonlinear hysteresis regulator completes the control chain by enforcing fast current tracking with adaptive switching boundaries. The resulting formulation is suitable for MATLAB/Simulink-based validation of harmonic compensation, power-factor correction, DC-link voltage regulation, and compliance with harmonic standards under nonlinear loading conditions.

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